King Fahd University of Petroleum & Minerals

Department of Information and Computer Science

Sample Solution

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	**Total**
CLO	1	1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3	2	2	2	
Max	5	5	5	5	5	5	5	5	4	4	4	3	3	3	3	3	3	10	10	10	**100**
Earned																					

Question 1: [5 Points]

What is the contrapositive of the conditional statement:

"If it is hot, I will go swimming"?

Question 2: [5 Points]

Show that if $p \oplus p$ is a tautology, a contradiction, or a contingency.

Question 3: [5 Points]

Are these system specifications consistent? Why? Why not?

a. If students can access the file system, then they can save new files.
b. Whenever the operating system is being upgraded, students cannot access the file system.
c. If students cannot save new files, then the operating system is not being upgraded.

Question 4: [5 Points]

Recall inhabitants of the island of knights and knaves. You encounter two people, A and B. A says "B is a knave" and B says "The two of us are both knights." Determine what A and B are.

Question 5: [5 Points]

Show that if $(\neg s \land (r \rightarrow s)) \rightarrow \neg r$ *is* a tautology, a contradiction, or a contingency.

Question 6: [5 Points]

Determine whether the compound proposition $(r \rightarrow s) \land (r \rightarrow \neg s) \land (\neg r \rightarrow s) \land (\neg r \rightarrow \neg s)$ is satisfiable.

Question 7: [5 Points]

The domain of the propositional function $Q(x)$ consists of the integers 1, 2, 3, 4, and 5. Express the statement

$\forall x\, ((x \neq 3) \rightarrow Q(x)) \lor \exists x\, \neg Q(x)$

without using quantifiers, instead using only negations, disjunctions, and conjunctions.

Question 8: [5 Points]

Translate the statement:

"Nothing is in the correct place and is in excellent condition"

into a logical expression using predicates, quantifiers, and logical connectives.

Question 9: [4 Points]

Let $P(x)$ be "x is a baby" and $Q(x)$ be "x is logical". Suppose that the domain consists of all people. Express the statement "Babies are illogical" using quantifiers; logical connectives; and $P(x)$ and $Q(x)$.

Question 10: [4 Points]

Suppose the domain of $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Express the statement $\exists x \forall y\, P(x, y)$ using disjunctions and conjunctions.

Question 11: [4 Points]

Rewrite the following statement so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

$\neg\exists y(Q(y) \land \forall x \neg R(x, y))$.

Question 12: [3 Points]

What is the cardinality of $\{a, \{a\}, \{a, \{a\}\}\}$?

Question 13: [3 Points]

If A and B are sets then what is the value of $A \cap (B -$

Question 14: [3 Points]

Is the following statement **TRUE** or **FALSE**?

$\{x\} \subseteq \{x\}$ and $\{x\} \in \{x\}$.

Question 15: [3 Points]

Is the following statement **TRUE** or **FALSE**?

If $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$ then $C \times B \times A = C \times A \times B$.

Question 16: [3 Points]

Is the following statement **TRUE** or **FALSE**?

If A and B are sets then $A - B \subseteq A$.

Question 17: [3 Points]

Is the following statement **TRUE** or **FALSE**?

If A, B, and C are sets then $(A - B) - C \subseteq A - C$.

Question 18: [10 Points]

Given the premises: $p \rightarrow (q \wedge r)$, $s \rightarrow r$, and $r \rightarrow p$.
show that $s \rightarrow q$.

RULES OF INFERENCE	
p ------ $\therefore\ p \vee q$	*(Addition)*
$p \wedge q$ ------ $\therefore\ p$	*(Simplification)*
p q ------ $\therefore\ p \wedge q$	*(Conjunction)*
p $p \rightarrow q$ -------- $\therefore\ q$	*(Modus ponens)*
$\neg q$ $p \rightarrow q$ -------- $\therefore\ \neg p$	*(Modus tollens)*
$p \rightarrow q$ $q \rightarrow r$ -------- $\therefore\ p \rightarrow r$	*(Hypothetical syllogism)*
$p \vee q$ $\neg p$ ------ $\therefore\ q$	*(Disjunctive syllogism)*
$p \vee q$ $\neg p \vee r$ ------ $\therefore\ q \vee r$	*(Resolution)*

Question 19: [10 Points]

Prove that the difference of an even integer minus an odd integer is odd. Use direct proof.

Question 20: [10 Points]

Using membership table, Prove that $(A_1 \cap A_2) \cup B = (A_1 \cup B) \cap (A_2 \cup B)$, for every sets A_1, A_2, B.